Application of Stimulated Electromagnetic Shock Radiation to the Generation of Intense Submillimeter Waves

S. SCHNEIDER AND R. SPITZER

Abstract—**A new synergistic effect, resulting in tunable high-frequency high-power narrow-band radiation, is described. The effect, called stimulated electromagnetic shock radiation (SESR), combines the Doppler shift in frequency of Compton backscattering of electromagnetic radiation from moving electrons with the formation of an electromagnetic shock front in a medium in which the Mach condition is satisfied. The underlying physical mechanism of SESR is contrasted with those of Cerenkov radiation and laser action. The characteristics of SESR and Cerenkov radiation are compared It is found that SESR is tunable in frequency and can be generated with much greater intensity and in a much narrower frequency band than Cerenkov radiation. The efficiency of conversion of electron energy into SESR is expected to be high (greater than 50 percent) under suitable conditions wherein the process is made self-limiting. The potential of SESR for generating submillimeter and far-infrared waves is discussed qualitatively.**

INTRODUCTION

WE CONSIDER the excitation of intense tunable submillimeter waves by means of a new effect. The effect, which to our knowledge has not as yet been observed, only calculated [1]–[3], involves the production of frequency-shifted narrow-band electromagnetic radiation by the scattering, in a suitable medium, of coherent electromagnetic waves from charged particles. The frequency of the scattered wave can be either upshifted or downshifted relative to that of the incident wave, but to make the discussion specific only the upward shift is considered here.

The general properties of the effect are described, its characteristics in dispersionless (weakly dispersive) media are summarized, and its potential for generating electromagnetic radiation in the submillimeter regime is discussed on the basis of general considerations.

GENERAL CONSIDERATIONS AND SESR IN A DISPERSIONLESS MEDIUM

The new effect, which we call stimulated electromagnetic shock radiation (SESR), involves two distinct and well-established phenomena. Each of the physical mechanisms underlying these phenomena is very general, and when conditions for both of them are satisfied simultaneously they will act synergistically to produce a basically new effect.

The first of the phenomena that underlie SESR is the Doppler shift in frequency of radiation scattered in vacuum from charged particles. When a charged particle of mass m and energy γmc^2 is accelerated in vacuum by an electromagnetic wave, the frequency ω of the radiated wave is shifted relative to the frequency ω_0 of the incident wave by γ^2 times a factor that depends on the collision (θ) and scattering (θ') angles:

$$\omega = \gamma^2(1 - \beta \cos \theta)(1 + \beta \cos \theta')\omega_0. \tag{1}$$

$\beta = u/c$ is the ratio of the speed u of the electron to that of light in vacuum and $\gamma = (1 - \beta^2)^{-1/2}$. The maximum frequency shift in vacuum occurs for backscattered waves from head-on collision, for which the scattered wave is upshifted in frequency by a factor of $4\gamma^2$,

$$\omega_{\max} = 4\gamma^2\omega_0. \tag{2}$$

The additional energy of the radiated wave is provided by the charged particle. This shift in frequency of backscattered radiation is the first result that bears on the new effect.

Different means of realizing the frequency upconversion by the Doppler shift of electromagnetic waves scattered from various configurations of electric charges have been summarized [4] and discussed further in their specifics [5] at the Second International Conference on Submillimeter Waves. In all these mechanisms, the upper bound on the upshifted frequency is given by (2).

The second phenomenon underlying SESR is also well known. It has long been recognized that basically new effects arise, in wave phenomena of completely different nature, when a material body moves in a medium with speed greater than that of the waves it produces in the medium. In the field of hydrodynamics the basic effect is a shock wave at supersonic velocities. For a charged particle moving through a medium with constant speed greater than a critical speed, the effect is Cerenkov radiation. Characteristic of such shock phenomena is the confinement of the energy of the generated wave within a conical region (Mach cone), a sharp peak in intensity on the surface of the Mach cone, and a sharply pronounced spatial asymmetry. This focusing by a medium of waves produced by a particle that satisfies the Mach condition is the second result that bears on the new effect.

Stimulated electromagnetic shock radiation is expected to occur when the Doppler shift in frequency of radiation scattered in a medium occurs with the Mach condition satisfied. The medium response along different points of the electron trajectory will then add coherently, resulting in a

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S. Schneider is at 26628 Fond du Lac Road, Palos Verdes Peninsula, CA 90274.
R. Spitzer is with the High Energy Physics Laboratory, Stanford University, Stanford, CA 94305.

radiation pattern confined in a conical region similar to the one that is obtained for Cerenkov radiation and intensifying near the surface of the cone.

Although both Cerenkov radiation and SESR are electromagnetic shock phenomena, and thus have certain features in common, the mechanisms for these two effects and their characteristics differ in essential respects. The mechanisms for both phenomena involve the coherent response of the medium to the field of the incident particle. That is, the beam is a source of an electromagnetic field that induces polarization charges and currents in the medium, and it is this polarization current which is the source of the radiation, both in the Cerenkov effect and in SESR. Symbolically, this mechanism is denoted, in obvious notation, by the sequence

$$\begin{aligned}(\rho_{\text{ext}}, \boldsymbol{j}_{\text{ext}}) &\rightarrow (\boldsymbol{E}_b, \boldsymbol{B}_b)\\ (\boldsymbol{E}_b, \boldsymbol{B}_b) &\rightarrow (\rho_{\text{ind}}, \boldsymbol{j}_{\text{ind}})\\ (\boldsymbol{j}_{\text{ind}}) &\rightarrow (\boldsymbol{E}_{\text{rad}}, \boldsymbol{B}_{\text{rad}}).\end{aligned} \tag{3}$$

Cerenkov radiation involves medium response to the beam field produced by the part

$$\boldsymbol{j}_c(\boldsymbol{x},t) = -e\boldsymbol{u}\delta(\boldsymbol{x} - \boldsymbol{u}t) \tag{4}$$

of the external current, where $\boldsymbol{u}$ is the velocity of the electron in the absence of the incident electromagnetic wave. This is a purely convective current, independent of the incident coherent electromagnetic wave, and is the source of a time-dependent *Coulomb* field. The phenomenon of SESR comes about because of medium response to the beam field produced by the part

$$\boldsymbol{j}_s(\boldsymbol{x},t) = -e\boldsymbol{v}(t)\delta(\boldsymbol{x} - \boldsymbol{u}t) \tag{5}$$

of the external current, where $\boldsymbol{v}(t)$ is the additional time-varying increment in the electron velocity stimulated by its interaction with the incident wave. The dependence of $\boldsymbol{j}_s$ on the parameters of the incident wave reflects the fact that SESR involves scattering of this wave from the incident electron, and $\boldsymbol{j}_s$ is the source of a *radiation* field. It is the role of the incident electromagnetic wave in first stimulating the incident electron to radiate that led us to propose the name SESR for this effect.

The differences in the characteristics of the two effects are indicative of the differences in their mechanisms. Cerenkov radiation has a broad frequency spectrum, being limited only by the dispersive properties of the medium, whereas the medium response in SESR will be sharply peaked about a specific frequency. Ultimately, the differences between the frequency characteristics of SESR and Cerenkov radiation come down to the essential differences in the Fourier spectra of the two parts (4) and (5) of the external (exciting) current.

The two effects are also distinguished by their relative dependence on the interaction distance L in the medium. Whereas Cerenkov radiation is known to depend linearly on L, SESR depends quadratically on L [2]. This difference can also be traced to the difference in their mechanisms [2]; the L^2 dependence arises from the stimulated nature of SESR and the overlap of the electron beam with the incident electromagnetic wave in the medium.

In both effects the power converted by the incident particle into radiation is equal to the rate of work done by the particle on the induced medium charges,

$$\frac{dW}{dz} = \frac{1}{u}\frac{dW}{dt} = \frac{1}{u}\int \boldsymbol{E}_b \cdot \boldsymbol{j}_{\text{ind}}\, d^3x. \tag{6}$$

$\boldsymbol{E}_b$ is the field of the incident particle and $\boldsymbol{j}_{\text{ind}}$ is the polarization current induced in the medium by this field. The incident particle's incremental energy loss into Cerenkov radiation is independent of the distance traversed in the medium because the magnitude of its time-dependent Coulomb field, being an intrinsic property of the particle, is independent of this distance. On the other hand, SESR is produced by the medium response to the particle's radiation field, whose amplitude is proportional to the interaction. The fact that the incident wave is in phase at different space–time points along the electron trajectory is decisive for the L^2 dependence of SESR.

The preceding argument for the existence and general properties of SESR is quite general. The formation of the Mach cone, when the incident particle exceeds a critical speed, follows from basically kinematical considerations, which assume only that when the medium electrons are excited by the *radiation* field of the incident particle, the field reradiated by the induced polarization currents will add coherently along its path and a shock front will build up. Thus the only extrapolation in the argument is that the medium will respond in a specific manner to the radiation field of the incident particle, given that it is known to respond in this manner to its time-dependent Coulomb field. It is therefore an extrapolation based on well-established properties of classical electrodynamics.

Indeed the simplest example that exhibits the effect, that of a dispersionless medium with a monochromatic wave incident either head on or parallel to the electron in the medium, is amenable to an exact solution of Maxwell's equations to leading order in the incident field and displays the expected characteristics [1]. This model of a polarizable medium, *but without the incident stimulating wave*, is the one originally considered by Frank and Tamm [6] in their explanation of Cerenkov radiation, so the characteristics of the two effects in a dispersionless medium can be compared. The use of a nondispersive medium also serves to eliminate the spread in frequency of the generated wave due to dispersion, and to isolate those frequency characteristics of the two phenomena that are due to the intrinsic spectra of the respective parts of the beam field exciting the medium to radiate.

The electromagnetic field of Cerenkov radiation in a dispersionless medium characterized by a constant susceptibility χ, $\varepsilon = 1 + \chi$, was derived by Tamm [7]. The component of the electric field along the trajectory of the electron is given, in terms of cylindrical coordinates (ρ,z), by

$$E_c(\rho,z,t) = \frac{2e}{\varepsilon u \gamma_s} \frac{\partial}{\partial t} \left[\frac{\theta(\tau)\theta(\gamma_s u\tau - \rho)}{Q}\right] \quad \text{(7a)}$$

$$\gamma_s = (\beta^2\varepsilon - 1)^{-1/2} \qquad \tau = t - \frac{z}{u}$$

$$Q = (\gamma_s^2 u^2 \tau^2 - \rho^2)^{1/2} \quad \text{(7b)}$$

$$\theta(x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x < 0. \end{cases}$$

The threshold for the occurrence of Cerenkov radiation is $\beta^2\varepsilon = 1$. The Cerenkov field is confined to within a conical region with the Mach angle (angle between electron trajectory and wave front) given by

$$\sin\phi = (\beta\sqrt{\varepsilon})^{-1}. \quad (8)$$

The field is sharply peaked in intensity on the surface of the cone, and decreases in intensity in the region backwards away from the inside edge of the cone. Formally, it is infinite on the surface of the cone ($Q = 0$). The origin of this infinity is well understood, and the singularity is known to be smoothed out by the absorption that must accompany the dispersive properties of any real medium.

The electric field of SESR in a dispersionless medium [1] is given by

$$\boldsymbol{E}_s(\rho,z,t) = \frac{2r_e\gamma_s \boldsymbol{E}_0}{\gamma\omega_0} \frac{\partial}{\partial t} \cdot \left[\frac{\theta(\tau)\theta(\gamma_s u\tau - \rho)}{Q} \cos(\alpha Q)\cos(\Omega_s t - Kz)\right] \quad \text{(9a)}$$

where $\boldsymbol{E}_0$ is the amplitude of the incident electromagnetic wave and

$$r_e = (e^2/mc^2) = 2.82 \times 10^{-13} \text{ cm}$$

$$\alpha = (\Omega_s\sqrt{\varepsilon}/\gamma_s c)$$

$$K = (\Omega_s/u)(1 + \gamma_s^{-2}) \quad \text{(9b)}$$

$$\Omega_s = \gamma_s^2\omega_0(1 + \beta\sqrt{\varepsilon}) \text{ for head-on collision.}$$

The field $\boldsymbol{E}_s$ is seen to share with the Cerenkov field E_c all the characteristics described in the preceding paragraph, but in addition it shows properties specific to SESR. It depends explicitly on $\boldsymbol{E}_0$, and its inverse dependence on the mass m of the incident charged particle indicates that SESR is an acceleration effect, as contrasted to Cerenkov radiation, which is a velocity effect independent of the inertial properties of the incident particle. The propagating-wave dependence of $\boldsymbol{E}_s$ exhibits explicitly the upshifted frequency Ω_s. We note that Ω_s becomes larger as the threshold value $\beta^2\varepsilon = 1$ is approached, provided, of course, that the threshold condition $\beta^2\varepsilon > 1$ is satisfied. This entails that for a given value of γ, the upshifted frequency can be made appreciably larger than $4\gamma^2\omega_0$ by suitable variation of the independent parameter ε. This qualitative difference between SESR and phenomena in which the maximum Doppler upshift in frequency is given by (2) demonstrates an essential difference in the physical content of the case where the Mach condition is satisfied and the case where the particle velocity is subcritical. The presence of the medium allows for supercritical conditions, which in turn affect qualitatively the frequency shift and provide for qualitative intensification of the electromagnetic radiation into a shock front.

Conversion of electron energy into Cerenkov radiation and into SESR is also markedly different. The energy converted in an incremental distance Δz into Cerenkov radiation in the frequency range $\Delta\Omega_s$ is given by the well-known expression [6]

$$\Delta W_c = \frac{e^2}{c^2}\left(1 - \frac{1}{\beta^2\varepsilon}\right)\Omega_s\,\Delta\Omega_s\Delta z. \quad (10)$$

This expression exhibits explicitly the fact that the intensity of Cerenkov radiation is weakest as the Mach cone opens up, that is, for $\beta^2\varepsilon$ near unity, corresponding to $\phi \simeq \pi/2$.

The expression for energy conversion into SESR is more complicated. An approximate relation, valid in the frequency range $10^{14} \lesssim \omega_0 \lesssim 10^{18}$ s^{-1} and for $\Delta z/c$ larger than Ω_s^{-1} (to allow time averaging of the energy density over a cycle of the frequency Ω_s), is

$$\Delta W_s \simeq \frac{3}{2}\left(\frac{\sigma_T\Delta W_L}{A}\right)\left(\frac{\gamma_s}{\gamma}\right)^2\left(\frac{\Omega_s}{\omega_0}\right)^2(\Omega_s\tau_\Delta)^2(\beta^2\varepsilon)^2\ln\left(\frac{R}{2\Delta z}\right) \quad (11)$$

where $\sigma_T = (8\pi/3)r_e^2$ is the Thomson cross section, ΔW_L is the macroscopic energy of the incident electromagnetic wave in the volume $A\Delta z$ ($\Delta W_L/A$ is the linear energy density),

$$\tau_\Delta \simeq (\Delta z/3u) \quad (12)$$

is a representative value of τ in the distance Δz [1], and R is the range of the electron in the medium. We note that although, as already indicated above, the kinematic nature of the shock of SESR is very similar to that of Cerenkov radiation, dynamically it is much more nearly like the hydrodynamic one than the Cerenkov one in that it is strongest at Mach 1, that is, for $\beta^2\varepsilon$ near unity, again corresponding to $\phi \simeq \pi/2$. Indeed it is seen from (11), (7b), (9b), and (8) that ΔW_s is inversely proportional to $\cos^{10}\phi$.

We find for the ratio of the two incremental conversion rates

$$\frac{\Delta W_s}{\Delta W_c} \simeq \frac{4\pi}{9}\frac{r_e(\Delta z)^2}{A\Delta z}\frac{\Delta W_L}{mc^2}\frac{\Omega_s}{\Delta\Omega_s} \cdot \left(\frac{\Omega_s}{\omega_0}\right)^2\left(\frac{\gamma_s}{\gamma}\right)^2\left(\frac{\gamma_s}{\beta}\right)^2(1 + \gamma_s^{-2})^3\ln\left(\frac{R}{2\Delta z}\right). \quad (13)$$

It is apparent that in general the expected absolute intensity of SESR can be estimated from the measured intensity of Cerenkov radiation under the same common conditions. It can be readily verified that for reasonable values of the adjustable parameters, including energy of the incident electromagnetic wave, the energy converted into SESR can be made significantly larger than that converted into Cerenkov radiation. Four factors allow for the design of a configuration wherein SESR is significantly larger than Cerenkov radiation: 1) the macroscopic energy content of the incident electromagnetic wave, which affects the stimulated process only; 2) the narrower frequency output of SESR; 3) the essentially different dynamical nature of the shock in SESR compared to Cerenkov radiation; and 4) the L^2 versus L dependence of the two effects together with the use of rare media (e.g., gases), which allows for a macroscopic interaction distance. Although expressions (11) and (13) are specific to weakly dispersive media, the known relation between the mechanisms for the two effects can, in principle, be exploited to obtain a corresponding relation for any experimental conditions. Basically, what is involved is scaling of the two effects according to the frequency components of $\boldsymbol{j}_s$ and $\boldsymbol{j}_c$. Moreover, it should be noted that, apart from factors of order unity and the logarithm, the other factors in (11), and hence also in (13), follow from general considerations of the mechanism for SESR or from dimensional arguments. Specifically, the dependence on the linear energy density $\Delta W_L/A$ is obvious once linear response to the incident field is assumed; the origin of σ_T has already been discussed in terms of the scattering of the incident wave from the electron; the time derivative in (9a), which brings down a factor of Ω_s, is general because the electric field is given as the derivative of the appropriate part of the vector potential; the factor $(\gamma_s/\gamma\omega_0)$ arises from the solution of the equation of motion of the electron in the field of the incident wave and the dynamics of the radiation process in the supercritical case, and together with the preceding factor of Ω_s accounts for $(\gamma_s/\gamma)^2(\Omega_s/\omega_0)^2$ in (11); the generality of the dependence on L^2 has already been indicated and, when combined with the number of oscillations per unit distance $(\Omega_s/2\pi u)$, expresses the cumulative dependence of the buildup of the wave energy in the shock front on this number via the factor $(\Omega_s\tau_\Delta)^2$ in (11).

The theoretical limit on the fractional amount of electron-beam energy that can be converted into SESR will be set by the limit in the distance over which the particle is in phase with the radiation, which in turn will be determined by the change in the particle velocity as it loses energy. Preliminary estimates indicate that in media such as a gas or a slow-wave structure evacuated in the region of the beam trajectory, for which the extrinsic energy losses due to effects other than SESR are relatively small, energy conversion into SESR will be self-limiting. That is, the limit in the acceptable change in particle velocity will be determined by this intrinsic energy-loss mechanism itself.

Before turning the SESR in slow-wave structures, we contrast briefly the laser and SESR mechanisms. The basis for the laser mechanisms is the excess of stimulated emission over stimulated absorption from a nonequilibrium distribution and an attendant release of energy stored in the medium. The SESR mechanism, in its simplest form, involves no change in the equilibrium distribution of level populations and no release of stored energy, but rather a *transfer* of energy from the electron into electromagnetic shock radiation, while maintaining in part the coherence of the incident electromagnetic wave.

SESR in Waveguides and Periodic Structures

Cerenkov radiation in waveguides has been widely analyzed theoretically and has been observed experimentally, most recently by Walsh *et al.* [8], who reference earlier work. The observation of this effect establishes that the kinematic condition for an electromagnetic shock wave can be satisfied in such structures.

The results of [8] also establish that collective effects due to bunching in the electron beam enhance the conversion rate by a large factor, of order 20, relative to that in single-particle experiments. The theoretical analysis [9] of this bunching effect shows that a necessary condition for the occurrence of enhanced Cerenkov radiation from bunched electrons is that the electrons have a larger than critical velocity, and hence that the shocklike nature of the effect persists when the radiation produced by different electrons adds in phase.

The generality of the mechanism underlying SESR and the similarity of the kinematic condition for the occurrence of the Cerenkov effect and of SESR make it apparent that the latter effect will manifest itself in waveguides and periodic structures under conditions where Cerenkov radiation is observable. Efficient energy transfer from the electron beam into SESR at a specific shock frequency in all such structures requires a match (strong overlap in both frequency and wave number integrations) between the dispersion relation governing formation of the shock wave and the relation that describes a specific mode of the medium. Because of the dependence of the shock frequency on the electron-beam velocity, incident-wave frequency, and medium properties, all of which can be varied continuously, the shock frequency is itself continuously variable, and hence such matching can in general be achieved by suitable variation of the available parameters.

An estimate of the expected magnitude of SESR relative to the Cerenkov radiation observed in a waveguide under the same common conditions can be obtained from relation (13), at least as to order of magnitude. For reasons discussed above and because the dispersionless case is simply a limiting case of the dispersive one, the factors in (13) that follow from general considerations are also expected to characterize this ratio for waveguides. Some caution must, however, be exercised in the choice of Δz when applying expression (13) under conditions where energy conversion into SESR is self-limiting, as this expression does not take into account the limit on Δz imposed by slowing down due to this mechanism. In general, the allowable Δz for a given configuration will necessarily depend on the degree of monochromaticity specified for the output. Preliminary estimates indicate, however, that, under optimum conditions, up to 50 percent or more of the electron-beam energy can be converted into SESR.

We note that in slow-wave structures the Mach condition can be satisfied for an electron moving in a void, so that collisional losses are diminished markedly or even eliminated entirely. This corresponds to the experimental conditions realized in the observation of Cerenkov radiation from collimated bunched electrons [8]. Under these conditions, energy conversion into SESR can almost certainly be made self-limiting. This has added importance in periodic structures, where the group velocity $\partial\omega/\partial\kappa$ can approach zero at the ends of the pass bands, i.e., when the Bloch wave numbers κ_i and components a_i of the cell size are related by $\kappa_i = n\pi/a_i$. The Mach condition can then be satisfied for low-energy electrons, for which very intense beams generated by relatively simple devices are available. The use of voided structures with low-energy electrons would eliminate the strong losses due to extrinsic energy-loss mechanisms that necessarily accompany transport of low-energy electrons in material media.

Conclusion

The generality of the mechanism underlying SESR indicates that SESR will be observable in waveguides and periodic structures under conditions where Cerenkov radiation is observable but with potentially much greater intensity, and with a tunable and sharper frequency output.

Although the above discussion focused on the use of counterflowing beams and on producing upshifted frequencies, all analytical results for a head-on collision can be modified to apply to a parallel-beam configuration by the substitution $\beta \rightarrow -\beta$, and, in general, the shock frequency can be upshifted or downshifted from the incident frequency for either configuration. However, antiparallel beams are convenient for an upshift, and parallel beams for a downshift. Generation of intense radiation in the submillimeter range and in that portion of the infrared spectrum where intense sources are presently unavailable is thus made possible either by an upshift from microwaves or a downshift from available intense infrared sources. Because of continuous tunability of SESR, the frequencies attainable by this mechanism are not limited to the discrete set of metastable transitions of a medium, as they are in the laser mechanism.

For example, an intense source of SESR at 16 μm can be produced by upshifting existing intense S-band sources interacting with counterflowing relativistic electrons in a polarizable gas with a susceptibility of order 10^{-4}. Alternatively, existing strong laser sources, e.g., CO_2 lasers, can be used to produce downshifted SESR by interacting with a parallel-flowing electron beam (not necessarily relativistic) in a void closely surrounded by a dielectric medium with $\varepsilon \simeq 1.6$. It is well known that an intense source of 16-μm radiation, which is not readily obtainable from conventional laser mechanisms, is important for uranium enrichment by isotope separation.

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